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13. ABSTRACT (Maximum 200 words) <p>This is the final report for research supported under AASERT Grant F49620-93-1-0505 during the period September 1, 1993 through August 31, 1997. This grant supplemented AFOSR Grant F49620-92-J-0014 by providing support for U.S. citizen graduate research assistants. This report summarizes the research achievements that have been made possible by this additional support during the period September 1, 1993 through August 31, 1997. The goal of the research has been to develop theory and engineering methods to facilitate the design of aerospace control systems with a robust tolerance to modeling uncertainty, including nonlinearity, disturbances and unmodelled dynamical perturbations. During the period of the AASERT grant, research effort was broadly focused on developing the theory of extending class of solvable robust control problems and on developing a theory to accommodate the issues that arise in going from experimental data to robust control designs. Significant progress was achieved in advancing the <i>Bilinear Matrix Inequality (BMI)</i> and the <i>Unfalsified Control</i> formulations of robust control problems. Missile and spacecraft design studies demonstrated the potential of these methods.</p>				
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Final Report:
AASERT SUPPLEMENT: ROBUST CONTROL METHODS
Grant Number F49620-93-1-0505

September 1, 1993 – August 31, 1997

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1 Research Objective

AFOSR Grant F49620-93-1-0505 supplemented AFOSR Grant F49620-92-J-0014 by providing support for U.S. citizen graduate research assistants. This report summarizes the research achievements that have been made possible by this additional support during the period September 1, 1993 through August 31, 1997. The goal of the research has been to develop theory and engineering methods to facilitate the design of aerospace control systems with a robust tolerance to modeling uncertainty, including nonlinearity, disturbances and unmodelled dynamical perturbations.

2 Accomplishments

During the period of AASERT Grant F49620-93-1-0505 from 1 September 1993 through 31 August 1997, three U.S. citizen graduate assistant received partial support from the grant:

Name	Dates	Status	Publications
Jason Q. Ly	9/93-5/95	PhD 5/95, now with Spectrum Astro, Manhattan Beach, CA	[1, 2, 4, 5, 6, 7, 8, 9, 10, 11]
P. Brugarolas	9/94-7/97	PhD expected 1999	[11]
S. Bohacek	7/97 - 8/97	PhD expected 1999	[3]

Highlights of the student research achievements include the PhD thesis of J. H. Ly [12] which advanced the new field of robust controller design via the use of bilinear matrix inequalities and included breakthrough results on " μ/k_m -synthesis control" which enabled the previously awkward step of curve-fitting to be bypassed. The AASERT grant also allowed progress to begin on the application of unfalsified control theory to a control of a guided missile [11] and, very recently, to the the attenuation of chaotic effects in certain difficult to control systems [3].

Eleven publications and one PhD thesis supported under AASERT Supplemental Grant F49620-93-1-0505 have either appeared, been submitted or are currently pending publication [1]–[12]. Areas of significant progress represented by these AFOSR supported publications include the following:

- Bilinear Matrix Inequality (BMI) control synthesis [1, 4, 5, 6, 7, 8, 9, 10]
- Robust Control with Non-Standard Uncertainties [3]
- Robust Aerospace Control Design [2, 11]
- PhD Thesis on K_m/μ -Synthesis [12]

Most of the theoretical developments embodied in the above listed recent AFOSR publications have been, or will soon be, implemented in software. Other concepts developed with AFOSR support played a central role in a the flexible satellite control design study [2]. The BMI control synthesis methods offer the promise of more precise and more reliable control of uncertain flexible space vehicles, such as the NASA Cassini spacecraft [2]. The effective and rapid transition from theory to practice has been facilitated by my on-going non-AFOSR-supported involvement with Dr. R. Y. Chiang in creating, and periodically upgrading, the MATLAB ROBUST CONTROL TOOLBOX, a robust control design software product published by The MathWorks and in use on more than 1000 government and industrial computer systems. Further details of several of the most significant achievements are elaborated below.

Bilinear Matrix Inequality (BMI) Synthesis

As shown in for example [5], a broad spectrum of robust control problems, including multimodel, decentralized, and reduced-order μ/K_m -synthesis problems, can be reformulated as Bilinear Matrix Inequality (BMI) Feasibility Problems. The BMI is an extension of the Linear Matrix Inequality (LMI) approach that has recently been found to be useful in formulating and solving a limited class of robust control problems, including state-feedback and full-order dynamical output feedback H_∞ control, μ/K_m analysis, simultaneous stabilization, gain-scheduling, and so forth. In particular, the BMI formulation offers the advantage of *simultaneously* handling all the foregoing types of specifications *as well as additional specifications not amenable to the LMI framework such as constraints on controller structure (e.g., decentralized "block-diagonal" control) and on controller order*. The BMI formulation also sheds new insight into the properties and limitations of existing robust control algorithms such as the μ/K_m -synthesis, indicating that the classical *DK*-iteration may not even produce locally optimal solutions.

Mathematically, the BMI is defined as follows:

Definition 2.1 (BMI Feasibility Problem) *Given real Hermitian matrices $F_{i,j} = F_{i,j}^T \in \mathcal{R}^{m \times m}$, for $i \in \{1, \dots, n_x\}$, $j \in \{1, \dots, n_y\}$. Define the matrix-valued bilinear function $F : \mathcal{R}^{n_x} \times \mathcal{R}^{n_y} \rightarrow \mathcal{R}^{m \times m}$:*

$$F(x, y) \triangleq \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} x_i y_j F_{i,j} \quad (2.0)$$

*Find, if they exist, real vectors $x = [x_1, \dots, x_{n_x}]^T \in \mathcal{R}^{n_x}$ and $y = [y_1, \dots, y_{n_y}]^T \in \mathcal{R}^{n_y}$ such that $F(x, y)$ is positive definite. This is called the **bilinear matrix inequality feasibility problem**.*

The global solution of such BMI's would resolve many of the major limitations the existing μ/K_m -synthesis theory for robust control design.

Also, as shown in the papers [5, 1], BMI's provide a natural formulation for the problem of optimal reduced-order H_∞ control synthesis. Likewise, while the controller structure constraints required in the synthesis of decentralized controllers have so far defied attempts to embed them in the LMI framework, these constraints are readily embedded within the BMI framework. Even more importantly, the BMI framework naturally handles the μ/K_m -synthesis with fixed-order generalized Popov multipliers [4].

While each of the two problems of solving for an optimal Popov multiplier $M(s)$ with the controller $K(s)$ fixed, and then solving for an optimal H_∞ controller $K(s)$ with the multiplier $M(s)$ fixed, is convex, the μ/K_m problem is not *jointly* convex in the multiplier and the controller. The upshot is that no guarantees of convergence to globally optimal values of $M(s)$ and $K(s)$ are possible. Indeed, solutions may not even be locally optimal. But, with the aid of the positive real lemma, we demonstrate in [5] that decentralized and reduced-order μ/K_m -synthesis control problems admit simple BMI formulations.

Our preliminary study [6] of the properties of the BMI feasibility problem indicates that it is possible to obtain local solutions which at least improve on existing alternating D - K synthesis techniques. However, the problem in general requires globally optimal solutions. In this regard, we find it encouraging that the global solution of a BMI has the simple interpretation that it is equivalent to finding the diameter of a collection of origin-centered ellipsoids in \mathcal{R}^N (cf. [6]: i.e., it is the diameter of a very simple, highly structured convex set. While the computation of the diameter of a convex set is not a convex problem, this result adds to our understanding of the challenge posed by robust control by distilling the key geometric properties of the underlying mathematical optimization problem and formulating the problem in a remarkably simple form.

And fortunately, not all BMI's lead to non-convex optimizations. Some reduce to a convex Linear Matrix Inequality (LMI) form for which globally solutions are readily computed. For example, the problem of μ/K_m -analysis in the presence of mixed real/complex uncertainties reduces to such a problem. The NASA Cassini spacecraft was one recent application of our research in LMI μ/K_m -analysis [2] — see Fig. 1.

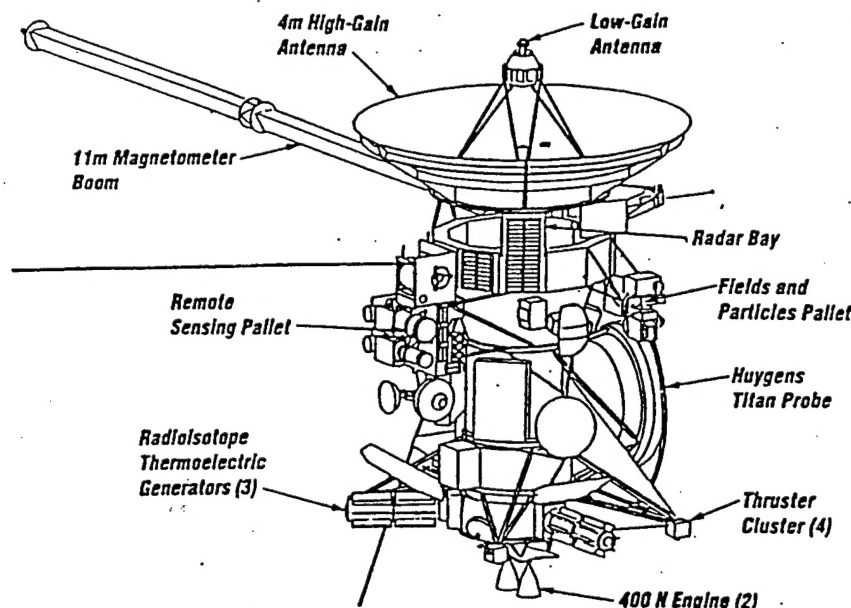


Figure 1: The Cassini spacecraft provided a recent application for the BMI/LMI theory [14].

Unfalsified Control Theory

Inspired by the “unfalsified model” concepts used in model validation and related system identification methods, but disappointed by their relative complexity and inherent conservativeness when used for control-oriented identification in conjunction with state-of-the-art robust control methods, a more direct “unfalsified control” approach was introduced by us in various papers supported by the primary AFOSR grant F49620-92-J-0014 culminating in the paper “The unfalsified control concept and learning” by M. G. Safonov and T. C. Tsao (*IEEE Trans. Autom. Control*, 42(6), June 1997).

Unfalsified control concept is a “model-free” approach to control. It works directly with input-output measurement data with the only model required being that of a parameterized class of candidate control laws. The central idea in our unfalsified control approach is that controller models can be “validated” against performance specifications directly from plant input-data without any need to identify or validate models of the plant itself. Furthermore, the computations required for direct “controller validation” are in general no more difficult than those required for plant model validation of the type in traditional system identification methods.

Thus, instead of attempting to enforce a somewhat artificial separation between modeling and

control design, our unfalsified control concept dispenses with plant models and uncertainty models altogether, focusing instead directly on the controller model and the implications of the available plant data regarding its capability to meet performance specifications. It replaces the conventional indirect two-step approach of (a) finding unfalsified plant models and (b) designing robust controllers. Our concept takes one directly from plant input-output data to control designs without the necessity of plant or uncertainty modeling. This is possible since all needed information about the plant is already in the plant input-output data — and this information turns out to be sufficient to validate control laws.

With AASERT support, these methods have recently been applied in a guided missile control design study [11]. The paper introduces a novel performance index which seems to produce better, simpler unfalsified controllers than had been possible previously.

3 Conclusions

With support from AASERT Grant F49620-93-1-0505, significant progress has been made in theory for reliable computation of robust controllers. In particular, significant progress in advancing BMI control synthesis theory resulted and the revolutionary new unfalsified control theory has been demonstrated in a missile control design study to be a useful aid in the design of adaptive/learning control systems.

4 List of AASERT Supported Publications

Journal Papers

- [1] K. C. Goh, M. G. Safonov, and J. H. Ly. Robust synthesis via bilinear matrix inequalities. *Int J. Robust and Nonlinear Control*, 6(9/10):1079–1095, November–December 1996.
- [2] J. H. Ly, R. Y. Chiang, K. C. Goh, and M. G. Safonov. LMI multiplier K_m/μ -analysis of the Cassini spacecraft. *Int J. Robust and Nonlinear Control*, to appear 1998. Special Issue on Robust Control Applications.
- [3] S. Bohacek and E. Jonckheere. Controlling chaos with chaotically varying linear quadratic control. *IEEE Trans. Autom. Control*, submitted August 1997.

Conference Proceedings

- [4] M. G. Safonov, J. Ly, and R. Y. Chiang. μ -synthesis robust control: What's wrong and how to fix it? In *Proc. IEEE Regional Conf. on Aerospace Control Systems*, pages 563–568. Thousand Oaks, CA, May 25–27, 1993. IEEE Press, New York.
- [5] M. G. Safonov, K. C. Goh, and J. Ly. Control system synthesis via bilinear matrix inequalities. In *Proc. American Control Conf.*, pages 45–49, Baltimore, MD, June 29–July 1, 1994. IEEE Press, New York.
- [6] K. C. Goh, L. Turan, M. G. Safonov, G. Papavassilopoulos, and J. Ly. Bilinear matrix inequality properties and computation methods. In *Proc. American Control Conf.*, pages 850–855. Baltimore, MD, June 29–July 1, 1994. IEEE Press, New York.

- [7] J. Ly, M. G. Safonov, and F. Ahmad. Positive real Parrott theorem with application to LMI controller synthesis. In *Proc. American Control Conf.*, pages 50–52, Baltimore, MD, June 29–July 1, 1994. IEEE Press, New York.
- [8] J. Ly, M. G. Safonov, and R. Y. Chiang. Real/complex multivariable stability margin computation via generalized Popov multiplier — LMI approach. In *Proc. American Control Conf.*, pages 425–429, Baltimore, MD, June 29–July 1, 1994. IEEE Press, New York.
- [9] K. C. Goh, J. H. Ly, L. Turan, and M. G. Safonov. μ/K_m -Synthesis via bilinear matrix inequalities. In *Proc. IEEE Conf. on Decision and Control*, pages 2032–2037, Lake Buena Vista, FL, December 14–16, 1994. IEEE Press, New York.
- [10] J. H. Ly, K. C. Goh, and M. G. Safonov. Multiplier K_m/μ -synthesis — LMI approach. In *Proc. American Control Conf.*, pages 431–436, Seattle, WA, June 21–23, 1995. IEEE Press, New York.
- [11] P. B. Brugarolas, V. Fromion, and M. G. Safonov. Robust switching missile autopilot. In *Proc. American Control Conf.*, Philadelphia, PA, June 24–26, 1998. Submitted 9/97.

Theses

- [12] J. H. Q. Ly. *A Multiplier Approach to Robust Analysis and synthesis*. PhD thesis, University of Southern California, August 1995. Supervised by M. G. Safonov.